The Biologist (Lima)

ORIGINAL ARTICLE / ARTÍCULO ORIGINAL

OPTIMAL AND INTELLIGENT SUBMARINE DESALINATOR BASED ON SMART SENSORS BY AMORPHOUS NANOSTRUCTURES

DESALINIZADOR SUBMARINO ÓPTIMO E INTELIGENTE BASADO EN SENSORES "SMART" MEDIANTE NANOESTRUCTURAS AMORFAS

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ABSTRACT

In this research, a smart system for a small submarine desalinator is proposed. According to getting optimal desalination of seawater that can be adapted and used by many denizens along the coast of Peru, which is located near the Pacific Ocean. This desalination submarine was designed using thermodynamic analysis to organize measurements of physical variables in seawater and during the desalination process (temperature, position, velocity, force). The desalination system was programmed autonomously to make its own decisions based on the physical variables measured by the submarine's sensors, including correlating these variables with theoretical models of chemistry and thermodynamics. These models, in turn, support the development of optimal, adaptive, and predictive systems for desalination. The proposed system can operate on the ocean surface and assist communities affected by water scarcity.

RESUMEN

En esta investigación, se propone un sistema inteligente para un pequeño desalinizador submarino. Con el objetivo de lograr una desalinización óptima del agua de mar, se propone adaptarlo y utilizarlo en la costa peruana, ubicada cerca del océano Pacífico. Este submarino desalinizador fue diseñado con aplicaciones de análisis termodinámico para organizar las mediciones de variables físicas del agua de mar y también durante el proceso de desalinización (temperatura, posición, velocidad, fuerza). Se programó la autonomía del sistema desalinizador para tomar sus propias decisiones sobre las variables físicas medidas por los sensores del submarino, inclusive la correlación de las variables medidas con modelos teóricos de química y termodinámica; que a su vez son un soporte para obtener sistemas óptimos, adaptativos y predictivos para la tarea de desalinización. El sistema propuesto puede operar sobre la superficie del océano, y ayudar a las comunidades humanas afectadas por la carencia de agua.

Palabras clave: desalinizador – nanoestructuras – optimización – sensores inteligentes

INTRODUCTION

Desalinator systems are quite necessary for desertic places, especially when there are many towns without access to get water for their domestic use (Hamed, 2017; Yahui *et al.*, 2023; Alenezi & Alabaiadly, 2025).

Consequently, it was designed an algorithm to simulate every response from the designed desalinator system (Åström & Hägglund, 2012), such as for example sun energy absorbance, temperature, pressure, humidity in the thermal chamber, as well as the physical variables from the dynamic of the desalinator system that are given by volume of condensed water, position, speed and vibration (Calderón *et al.*, 2022).

Moreover, the algorithm designed for the simulations was adapted for the experimental analysis to meet instrumentation requirements, such as the selected microcontroller to control all processes on the designed desalinator. It was crucial to recognize that a short response time and high robustness of every sensor are necessary. For this reason, the targets were achieved due to the sensors (designed smart sensors) being based on nanostructures (Calderón *et al.*, 2022), this means that the main control algorithm of the designed system will get more time to execute intricate tasks (Lei *et al.*, 2007) in order to achieve an optimal desalinization; ergo, the proposed submarine was based on simple and standard designs, but improved by advanced sensors due to achieve good performance in the navigation because of optimal dynamical analysis as it described on paragraphs above (Analog Devices, 1999; Ranjna *et al.*, 2023; Yunhwan *et al.*, 2025).

The Fig. 1 depicts part of the setup for the desalinization physical analysis, in which is represented the desalinator system, this is composed by its collector "A" (light blue cylinder), also by its depositor "W" (a cone with own filter), as well as the main ocean water depositor "B" (the bigger cylinder in the system). Hence, the sun "S" heat achieved by the desalinator system is the main energy to get the water condensation over "A".
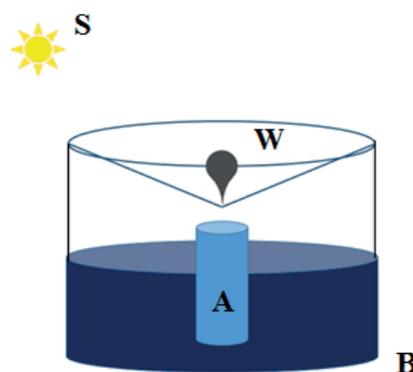

Figure 1. Scheme of a general desalinator based on condensation. W = a cone with own filter. B = the bigger cylinder in the system. S = sun. A = water condensation.

In fact, it was studied and analyzed some techniques according to design a mechatronic system to deal with the desalinization problem, for which the designed model also has the possibility to be a support for users on the sea and outside, as well as to support guiding users to find optimal roads in the ocean while getting communication with different users (inside the sea and outside) in concurrent to desalinate water and storing inside itself. Hence, it is an objective to propose an analytical mathematical model to

correlate the dynamic system of the mechatronic design, a robust energetic system, and an optimal communication system, which is explained and detailed in the following chapter.

Therefore, in this research are proposed some systems to solve this task, which is supported by a previous understanding of the problematic "desalinate ocean water for domestic use". Thus, it was possible to design a mathematical model based on the correlation between thermodynamic analysis with Modulating Functions according to obtain a robust model to describe the desalinization process.

MATERIALS AND METHODS

The designed system can be explained by the dynamics and thermodynamics of the proposed smart desalinator. Most of the measured physical variables are given by first order models (That is based on their linear operating work chosen for this research), hence, it was necessary to study that equations, such as given by the equation (1) in Laplace domain "S", in which "T" is the temperature variable, "U" is the input excitation variable, "" is the gain of the system (Åström & Hägglund, 2004; Bistak *et al.*, 2023), "" is the response time and "" is its time delay (Vajta, 2000; Idrees, 2017).

$$\frac{T(S)}{U(S)} = \frac{K_p}{\Upsilon S + 1} e^{-LS} \quad (1)$$

The following equation (2) is consequently the inverse Laplace transformation on the equation (1), in which it is considered the function "", because of the delay analysis. In addition that "" helps to get information of the physical processes delays during the water desalinization (during the condensation process) (Sherman *et al.*, 2023).

$$\Upsilon \frac{dT(t)}{dt} + T(t) = K_p U(t - L) \, \mathrm{y}(t - L) \quad (2)$$

For the operating work of the time "t" between 0 to "L", it is proposed the equation (3), which is the result from the equation (2).

$$\Upsilon \frac{dT(t)}{dt} + T(t) = 0 \quad (3)$$

In the context that "U" is zero under its time domain, because the system is not under its stimulation, such as a consequence the previous equation (3) is reduced to the equation (4).

$$T(t) = T_0 \quad (4)$$

Whereas, for the context t >= L, it was obtained the equation (5).

$$\Upsilon \frac{dT(t)}{dt} + T(t) = K_p U \quad (5)$$

Correlating the solutions from previous equations, with the Fourier heat transfer model, which is given by the equation (6), where "q" is the heat, "k" is the thermal resistivity, and "T" is the temperature in dependence on the geometry and heat propagation road. Otherwise, it was possible to get good estimations of the condensed water mass.

$$q = -k \, \nabla T \quad (6)$$

The equation (7) shows an expanded model from the equation (6).

$$q = -k \left(\frac{dT}{dx}, \frac{dT}{dy}, \frac{dT}{dz} \right) \quad (7)$$

Therefore, by the described equations above, it is possible to estimate the heat achieved by the designed desalinator (Medina, 2009). As well as, the following equations propose the temperature of the desalinator surface, where it is obtained the condensed water after the desalination process. The heat model, by heat intensity "I", in dependence on the frequency "w", speed of light "C", Planck constant "", universal constant "K", temperature "T", which is proposed by the equation (8) according to get a general model of heat transfer by radiation (Landau & Lifshitz, 1959; Feynman *et al.*, 1962).

$$I = \frac{\hbar \, w^3}{\pi^2 C^2 \left(e^{\frac{\hbar w}{KT}} - 1 \right)} \quad (8)$$

Furthermore, it is known the equation (9) (Landau & Lifshitz, 1959; Feynman *et al.*, 1962).

$$\frac{Q}{V} = \int_0^\infty \frac{I}{C} dw \quad (9)$$

From the equation (8), it is proposed the equation (10).

$$x = \frac{\hbar w}{KT} \quad (10)$$

It is achieved the equation (11) from the equation (10).

$$\frac{KT}{\hbar} dx = dw \quad (11)$$

The equation (12) is achieved replacing the equation (8) in the equation (9).

$$\frac{Q}{V} = \int_0^\infty \frac{\hbar \, w^3}{\pi^2 C^3 \left(e^{\frac{\hbar w}{KT}} - 1 \right)} dw \quad (12)$$

From equations (10) and (11) in equation (12), it is obtained the equation (13).

$$\frac{Q}{V} = a \int_0^\infty \frac{x^3}{e^x - 1} dx \quad (13)$$

In which, it is proposed the equation (14).

$$a = \frac{(KT)^4}{\pi^2 (\hbar C)^3} \quad (14)$$

Therefore, from the equation (13), it is organized the equation (15).

$$\frac{Q}{V} = a \int_0^\infty x^3 (e^x - 1)^{-1} dx \quad (15)$$

From the equation (15), it is expanded to obtain the equation (16).

$$\frac{Q}{V} = a \int_0^\infty x^3 (e^{-1x} + e^{-2x} + e^{-3x} + \cdots) dx \quad (16)$$

Moreover, the equation (16) is reduced to the equation (17).

$$\frac{Q}{V} = a \int_0^\infty x^3 \sum_{n=0}^\infty e^{-nx} dx \quad (17)$$

Nevertheless, it was organized the equation (18) from the previous equation (17) (Landau & Lifshitz, 1959; Feynman *et al.*, 1962).

$$\frac{Q}{V} = a \sum_{n=1}^\infty \int_0^\infty x^3 e^{-nx} dx \quad (18)$$

As well as, it is known the equation (19) in order to reduce the equation (18).

$$\int_0^\infty e^{-nx} dx = -\frac{1}{n}\left(\frac{1}{e^\infty} - \frac{1}{e^0}\right) \quad (19)$$

Therefore, it was achieved the equation (20).

$$\int_0^\infty e^{-nx} dx = n^{-1} \quad (20)$$

Derivative by "n" on the equation (20), it was obtained the equation (21).

$$\frac{d}{dn}\left(\int_0^\infty e^{-nx} dx\right) = \frac{d}{dn}(n^{-1}) \quad (21)$$

Derivative by "n" again to achieve the equation (22).

$$\frac{d}{dn}\left(\int_0^\infty (-x) e^{-nx} dx\right) = \frac{d}{dn}((-1)n^{-2}) \quad (22)$$

Derivative by "n" one more time again, according to achieve the equation (23).

$$\frac{d}{dn}\left(\int_0^\infty (-x)(-x) e^{-nx} dx\right) = \frac{d}{dn}((-1)(-2)n^{-3}) \quad (23)$$

In fact, it was obtained the equation (24).

$$\int_0^\infty (-x)(-x)(-x) e^{-nx} dx = (-1)(-2)(-3)n^{-3} \quad (24)$$

As well as, it was reduced to the equation (25).

$$\int_0^\infty x^3 e^{-nx} dx = 6n^{-4} \quad (25)$$

Equation (25) in (18), it was obtained the equation (26).

$$\frac{Q}{V} = a \sum_{n=1}^\infty \frac{6}{n^4} \quad (26)$$

Then, the equation (27) is consequently from the equation (26)

$$\frac{Q}{V} = 6a\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\right) \quad (27)$$

Even though, the equation (28) generalizes a model to achieve the heat by radiation that was useful to get condensed water from the desalinator system that was designed for this research (Landau & Lifshitz, 1959; Feynman *et al.*, 1962).

$$\frac{Q}{V} = \sum_{n=1}^\infty \frac{6}{n^4} \frac{(KT)^4}{\pi^2 (\hbar C)^3} \quad (28)$$

As well as, the dynamic model (mathematical analysis) for the desalinator system was determined by generalized Lagrange that is given by the following equation.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial x}\right) = \left(\frac{\partial L}{\partial r}\right) \quad (29)$$

In which "L" depends on the kinetic and potential energy of the main system (desalinator), in spite of it was possible to find solutions through the correlation of the desalinator dynamics with the thermodynamics effects. Hence, in the Fig. 2, it is depicted the desalinator model prototype of this research "DS".

The data communication is received and transmitted through its antenna "A", the main control algorithm executed by the controller "C", the sensor system for this desalinator designed is represented by "S", the condensed water chamber "T", the power subsystem "A", where are stored the rechargeable batteries (hybrid model due to part of them are composed with small batteries based on nanostructures for this research), even though part of the recharged energy was achieved by small sun panels (based on nanostructures) during the condensed process; therefore, there were controlled the propellers "P1 and P2" owing to get a controlled movement for the desalinator prototype also in the ocean (Wang *et al.*, 2021).
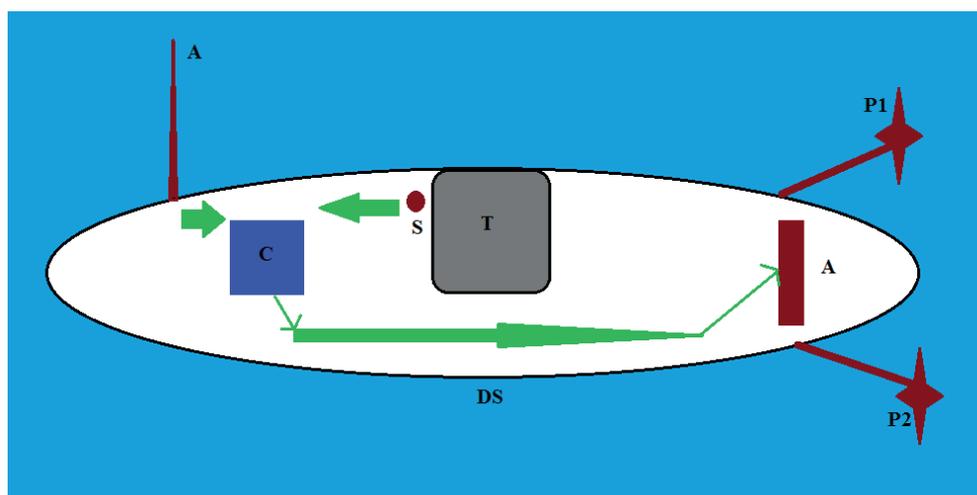
Figure 2. Desalinator prototype scheme. DS = Desalinator model prototype of this research. A = antenna. C = controller. S = sensor system. T = condensed water chamber. P1 and P2 = propellers.

It was analized a second order differential equation for the dynamic interpretation of the desalinator, which is nonlinear model (Medina, 2010). Notwithstanding, from the second order differential equation (30) it was deduced the nonlinear model by the coefficients "a_1, a_2, a_3, and b_1" and response variable "$y(t)$" depending on its input variable "$u(t)$" (Santana, 2023; Strang & Herman, 2025).

$$a_1 \frac{d^2 y(t)}{dt^2} + a_2 \frac{dy(t)}{dt} + a_3 y(t) = b_1 u(t) \qquad (30)$$

For which, the coefficient "a_2" was obtained in the static response analysis that is given by the equation (31), where this coefficient got proportionality in dependence on "h".

$$a_2 = h \frac{dy(t)}{dt} \qquad (31)$$

There were not deformations on the main system and there is considered an impulse as the main input excitation signal during a short proposed steady state, therefore it was achieved the following equation (32).

$$a_1 \frac{d^2 y(t)}{dt^2} + h \left(\frac{dy(t)}{dt} \right)^2 = 0 \qquad (32)$$

As well as, it was proposed the equation (33).

$$v = \frac{dy(t)}{dt} \qquad (33)$$

Hence, the equation (34) was obtained as a consequence to replace the equation (33) in the equation (32).

$$\frac{a_1}{h} dv(t) = -v^2 dt \qquad (34)$$

The equation (35) is a reduction from the equation (34).

$$-\frac{a_1}{h} \frac{dv(t)}{v^2} = dt \qquad (35)$$

Looking for the integral in the equation (35), it was obtained the equation (36).

$$-\frac{a_1}{h} \int_{v_0}^{v} \frac{dv(t)}{v^2} = \int_{v_0}^{v} dt \qquad (36)$$

Hence, the equation (37) is the reduction of the equation (36).

$$\frac{a_1}{h} \left(\frac{1}{v} - \frac{1}{v_0} \right) = t - t_0 \qquad (37)$$

From which, it was obtained the equation (38), where the initial speed "" of the desalinator designed is not null, however, the equation helps to validate the circumstances when it returns to get static equilibrium.

$$v = \frac{1}{\frac{h}{a_1}(t - t_0) + \frac{1}{v_0}} \qquad (38)$$

Furthermore, from the equation (38), it was possible to deduce the equation (39) according to estimate the maximal distance traveled till the desalinator will get its static equilibrium again.

$$\int_{v_0}^{v} dy = \int_{v_0}^{v} \frac{dt}{\frac{h}{a_1}(t - t_0) + \frac{1}{v_0}} \qquad (39)$$

Therefore, it was analyzed and detailed the problematic to achieve an advanced and robust mathematical model to design a mechatronic system to be a support for users as desalinator and support on the ocean and outside.

However, it was necessary to evaluate the performance of the designed mathematical models (which were studied by equations described in paragraphs above) by simulations and experimental results. It means, in the following chapter are described results of the adaptive algorithms designed for the simulations results and the consequence applications of the experiments based on the hardware mechatronic design of the proposed desalinator.

Ethic aspects: This article has not ethical conflicts in the proposed research, which was cited every bibliographic reference for every analysis described.

RESULTS

The results achieved in this research are focused on the thermal effects of the designed desalinator, aside from its dynamic performance, it means that the desalinator system needed maximal 25 Watts (average around 10

Watts) during 60 minutes of heat absorbance, in order to obtain approximately 225 mL of condensed water, which is showed on the Fig. 3. Hence, the proposed desalinator system, which looks like a small submarine optimized its advanced control system because of its dynamic response on the sea and outside, as well as its optimal thermodynamic response helped to condensate sea water in concurrent with other tasks that the desalinator was solving, such as for example its internal communication systems by Infrared (IR) or external communication with users by Radio Frequency (RF).

The condensation sea water needed a sophisticated coordination analysis between the control activities of the desalinator system with the integration of smart sensors prepared for the thermal responses/tasks, moreover the strategical characteristics of the desalinator system based on the maximal possibilities to get sun energy over the condensation cabin with the optimal heat absorbance, which depended on the nano materials covering over the condensation cabin.
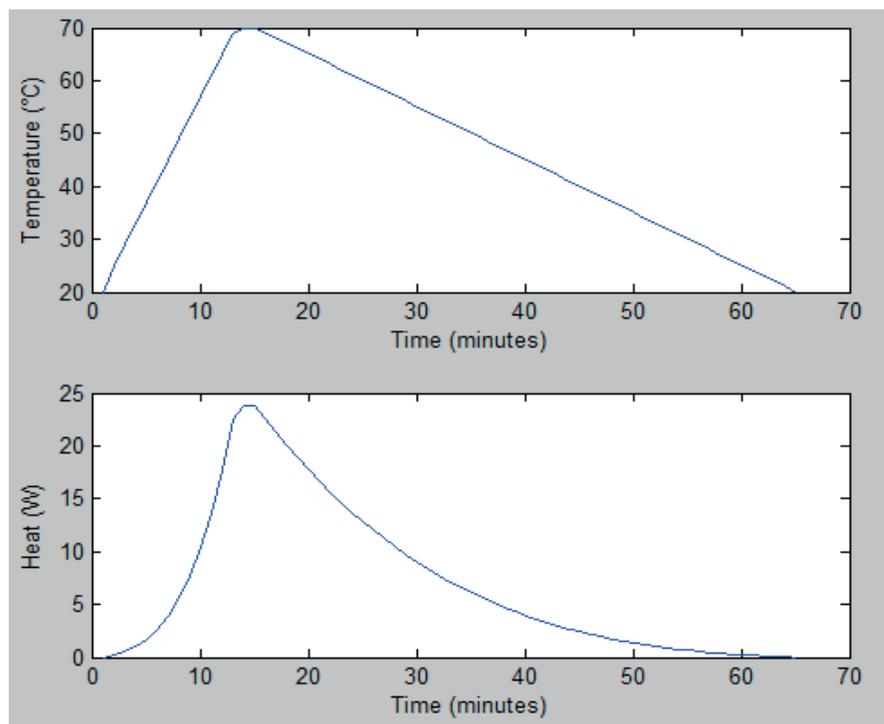

Figure 3. Heat and temperature inside the designed desalinator system.

Therefore, the heat absorbance by radiation, the optimal measured variables of humidity, sea water level, temperature by sensors based on nanostructures that were designed for this desalinator, supported to achieve optimal condensation due to short response time of the sensors based on nanostructures, which also was correlated with faster data

communication (by wireless, using IR) between internal modules in the desalinator system.

In fact, the capacity to organize the internal process of the small submarine coordinated with the smart sensors based on nanostructures gave the chance that the main

control system could not be saturated by redundant tasks (because the smart sensors reduce the processing task), then the condensation process was enough optimized by the main control system, by the smart sensors and the optimal covering nano materials to improve the heat absorbance. For this reason, the water condensed volume also tended to get linear response on dependence of the geometrical proportion of the designed desalinator.

For the dynamical analysis results of the designed desalinator, it was necessary to coordinate the integration activities between the smart sensors based on nanostructures to measure its displacement, its impulse force, its speed. Hence, the main control system identified online the

physical parameters for the optimal displacement of the desalinator system depending on the correlation between dynamic physical laws of the small submarine with Modulating Functions parameters achieved online, which was a good information proportionate optimal trajectories for the submarine.

The Fig. 4, shows the response variables "speed and position" of the designed desalinator, while it is moving inside and outside the sea. The previous equations above helped to design the adaptive algorithms to get optimal estimations of the main physical variables to correlate the condensation task with the desalinator displacements on the sea, as well as for outside tasks.
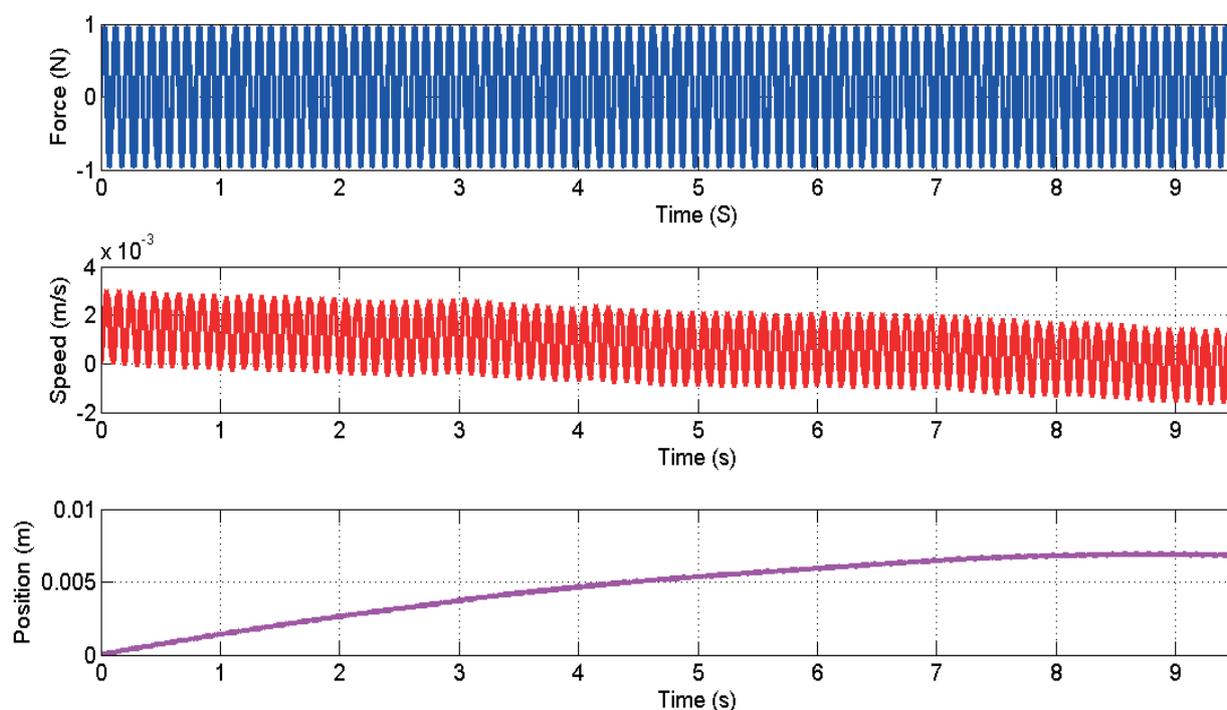

Figure 4. Response variables of the desalinator dynamics.

DISCUSSION

It had been designed a small automate that can displace itself inside the sea, by the dynamical analysis of the external forces around it. The designed system has the possibility to get smart responses due to most of its physical variables are measured by sensors based on nanostructures, it means there were obtained short response time and high robustness in comparison of the traditional electromechanical sensors (Texas Instruments, 2017). Nevertheless, the operating work for every physical variable was not enough according to evaluate the performance of the designed smart submarine under longer distances (Nafey & Safwat, 1988).

For the context of the designed position sensors, it was worked by the IR sensors, which helped to get estimations of bodies around the designed submarine, however, there were problems during the optimal movement when the IR signal did not find appropriated surfaces for its optimal distance estimation, hence, it was complemented the distance detection by ultrasound sensors based on nanostructures (Yoon *et al.*, 2022).

In fact, the communication between the submarine prototype with the external medium, as well as receiving orders/tasks by the main user was optimal under delimitated range of work, which depends on faster and robust response

from its sensors, that is quite important due to coordinate the main task of the smart submarine owing to be helper for the external user, such as for example to give the ubication and image information inside the sea (Bistak *et al.*, 2023). It means, the optimal coordination between internal modules of the designed desalinator with its smart sensors based on nanostructures, moreover with nano covering material to improve heat absorbance, also a good energy and communication system (by IR and RF) got as a consequence a sophisticated desalinator that achieves sea water condensation and optimal movement on the sea and outside owing to be a good support for users.

ACKNOWLEDGEMENT

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